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Note

On the Evaluation of Double-Folding, Heavy-Ion Interaction Potentials by Fourier Transformation Methods

It has recently been shown [1] that folding potentials are quite successful in describing heavy-ion scattering data. The folding model requires taking the three-dimensional folding product between the nuclear density of the target and an effective nucleon–nucleon interaction. This product is then to be folded into the projectile density. The resulting six-dimensional folding integral is most conveniently evaluated in Fourier space. In [1] spherical nuclear density distributions are generated numerically and subsequently their Fourier–Bessel transforms are also calculated numerically. These steps can be simplified if the density is represented by a leptodermous distribution for which the Fourier–Bessel transform is an algebraic expression of elementary functions.

The folding model with fixed spherical nuclear densities is physically only reasonable if target and projectile densities overlap at most with their exponential tails. It is therefore only the tail of the distribution which really matters in the folding model. The actual density may then be substituted by a leptodermous function whose radius and diffuseness parameters were fitted to reproduce the tail of the density. Compare [2, 3] for an accurate determination of the equivalent sharp radius R (in these papers called the dms radius) and the Helm-model skin thickness t_H of a given distribution. For light $N = Z$ nuclei the density parameters can be deduced from experimental proton densities [2] (not charge densities!). Alternative methods to obtain experimental mass distributions are discussed in [1].

Leptodermous distributions may be represented either by a Woods–Saxon shape $\rho_0(1 + \exp\{(r - R_{1/2})/a_F\})^{-1}$ with the parameters ρ_0, $R_{1/2}$, and a_F or the folding product $\rho_0 \Theta(r - R) * Y$ with

$$\Theta = 1, \quad \text{inside the nucleus}, \quad r \leqslant R,$$
$$= 0, \quad \text{outside the nucleus}, \quad r > R,$$

and

$$Y = (4\pi a^2 r)^{-1} e^{-r/a}.$$

The radius parameters are related by

$$R = [R_{1/2}(R_{1/2}^2 + \pi^2 a_F^2)]^{1/3} + \mathcal{O}(e^{-R/a_F}) \approx R_{1/2}[1 + (\pi^2/3)(a_F/R_{1/2})^2]$$

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and the various thickness parameters can all be expressed in terms of Myers' diffuseness parameter b [4]

$$a^2 = b^2/2, \qquad a_F^2 = 3b^2/\pi^2, \qquad t_H^2 = 6.45b^2.$$

If the density distribution is represented by Woods–Saxon shapes, the potential

$$V(r) = \rho_P * V_{M3Y} * \rho_T = \rho_P * \left(\sum_{i=1}^{3} v_i Y_i\right) * \rho_T$$

can be obtained by Fourier techniques [5], which yield

$$V(r) = \sum_{i=1}^{3} \frac{v_i}{2\pi^2} \int_0^\infty q^2 j_0(qr)\, F[\rho_P]\, F[Y_i]\, F[\rho_T]\, dq \tag{1}$$

with the Fourier transforms [6]

$$F[\rho_P] = 3A_P \frac{a_1}{R_1} \frac{1}{(R_1/a_1)^2 + \pi^2} \left[\frac{\pi a_1 q \operatorname{ctgh} \pi a_1 q \sin qR_1 - qR_1 \cos qR_1}{a_1^2 q^2 \sinh \pi a_1 q} \pi \right.$$
$$\left. - 2 \sum_{n=1}^{\infty} (-)^n \frac{n}{(n^2 + a_1^2 q^2)^2} e^{-nR_1/a_1} \right],$$

similarly $F[\rho_T]$, and

$$F[Y_i] = (1 + \alpha_i^2 q^2)^{-1}.$$

Here a_1 and R_1 are the projectile a_F and $R_{1/2}$ values. The parameters of the M3Y effective interaction [1] are

$$v_1 = 1571 \text{ MeV fm}^3, \qquad \alpha_1 = 0.25 \text{ fm},$$
$$v_2 = -1716 \text{ MeV fm}^3, \qquad \alpha_2 = 0.40 \text{ fm},$$
$$v_3 = -262 \text{ MeV fm}^3, \qquad \alpha_3 = 0.0 \text{ fm}.$$

Note that the integrand in (1) is oscillating with wave lengths π/R_P, π/R_T, π/r, which requires a rather small step size in the numerical evaluation of the integral, in particular for large r. In this case however, the Fermi functions are sometimes approximated by the first term of the series

$$\rho = \rho_0 \sum_{n=1}^{\infty} e^{-n(R-r)/a} \tag{2}$$

valid for $r > R$. Substitution of

$$F[\rho_0 e^{(R-r)/a}] = \rho_0 e^{R/a} \frac{8\pi a^3}{(1 + a^2 q^2)^2} = 6A \frac{a}{R} \frac{e^{R/a}}{(R^2/a^2 + \pi^2)(1 + a^2 q^2)^2}$$

into (1) yields by contour integration [6]

$$V(r) \underset{r\to\infty}{\approx} \frac{9A_1A_2}{2\pi r}\frac{a_1}{R_1}\frac{a_2}{R_2}\frac{e^{R_1/a_1}}{(R_1/a_1)^2+\pi^2}\frac{e^{R_2/a_2}}{(R_2/a_2)^2+\pi^2}\sum_{i=1}^{3} v_i B_i(r) \tag{3}$$

with

$$\begin{aligned} B_i(r) = {} & \left(1-\frac{a_2^2}{a_1^2}\right)^{-2}(a_1^2-\alpha_i^2)^{-1}\left[\frac{r}{a_1}+\frac{4}{1-a_1^2/a_2^2}+\frac{2}{1-a_1^2/\alpha_i^2}\right]e^{-r/a_1} \\ & + \left(1-\frac{a_1^2}{a_2^2}\right)^{-2}(a_2^2-\alpha_i^2)^{-1}\left[\frac{r}{a_2}+\frac{4}{1-a_2^2/a_1^2}+\frac{2}{1-a_2^2/\alpha_i^2}\right]e^{-r/a_2} \\ & + 2\left(1-\frac{a_1^2}{\alpha_i^2}\right)^{-2}\left(1-\frac{a_2^2}{\alpha_i^2}\right)^{-2}\alpha_i^{-2}e^{-r/\alpha_i}. \end{aligned}$$

Then B_i becomes indefinite if two a parameters are equal. Its value is then determined by De l'Hospital's rule.

In evaluating the folding product expansion (2) was implicitely used also for $r < R$, where it is not convergent. Therefore (3) is only asymptotically valid. It is seen in Figs. 1 and 2 that it is a rather poor approximation for (1) for all r values of interest. It can be shown that keeping higher order terms in (2) even worsens the approximation.

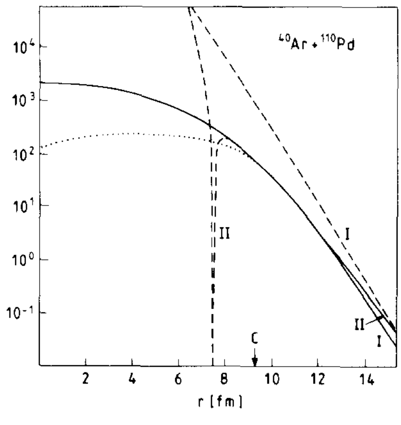

FIG. 1. Heavy-ion interaction potential (nuclear part only) for the system ^{40}Ar + ^{110}Pd. The nuclear density was represented by a Woods–Saxon shape (I) and a folding product (II). The asymptotic forms (3) and (4) are shown by (---). A smooth extrapolation (···) of (4) towards smaller radii is to be used for adiabatic collisions. The contact point of the equivalent sharp surfaces is indicated by C on the abscissa.

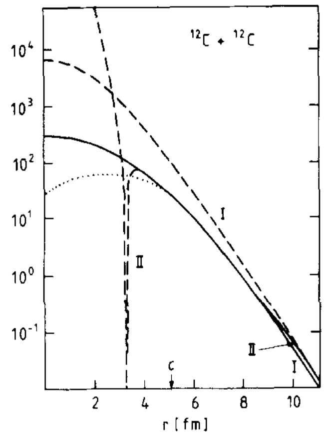

FIG. 2. Same as Fig. 1 for the system $^{12}C + ^{12}C$.

If the densities are represented by the folding product $\rho_0 \Theta * Y$, integral (1) can be evaluated directly by contour integration [5] for $r \geqslant R_1 + R_2$

$$\begin{aligned} V(r) &= A_1 A_2 \frac{9}{2\pi^2} \sum_{i=1}^{3} v_i \int_0^\infty j_0(qr) \frac{j_1(qR_1)}{qR_1} \frac{j_1(qR_2)}{qR_2} \\ &\quad \times (1 + a_1^2 q^2)^{-1} (1 + a_2^2 q^2)^{-1} (1 + \alpha_i^2 q^2)^{-1} q^2 \, dq \\ &= A_1 A_2 \frac{9}{4\pi r R_1 R_2} \sum_{i=1}^{3} v_i C_i(r) \end{aligned} \tag{4}$$

with

$$\begin{aligned} C_i(r) = &\left(1 - \frac{a_2^2}{a_1^2}\right)^{-1} \left(1 - \frac{\alpha_i^2}{a_1^2}\right)^{-1} e^{-r/a_1} \prod_{j=1,2} \left(\frac{a_1^2}{R_j^2} \sinh \frac{R_j}{a_1} - \frac{a_1}{R_j} \cosh \frac{R_j}{a_1}\right) \\ &+ \left(1 - \frac{a_1^2}{a_2^2}\right)^{-1} \left(1 - \frac{\alpha_i^2}{a_2^2}\right)^{-1} e^{-r/a_2} \prod_{j=1,2} \left(\frac{a_2^2}{R_j^2} \sinh \frac{R_j}{a_2} - \frac{a_2}{R_j} \cosh \frac{R_j}{a_2}\right) \\ &+ \left(1 - \frac{a_1^2}{\alpha_i^2}\right)^{-1} \left(1 - \frac{a_2^2}{\alpha_i^2}\right)^{-1} e^{-r/\alpha_i} \prod_{j=1,2} \left(\frac{\alpha_i^2}{R_j^2} \sinh \frac{R_j}{\alpha_i} - \frac{\alpha_i}{R_j} \cosh \frac{R_j}{\alpha_i}\right). \end{aligned}$$

Note that $V(r)$, defined in (1), is not an analytic function of r. Therefore (4) should not be continued analytically to $r < R_1 + R_2$. The figures show that it deviates drastically from (1) in the interior. If any two of the three parameters a_1, a_2, α_i coincide, $C_i(r)$ has to be determined by De l'Hospital's rule. The parameters A_j, R_j, and a_j, $j = 1, 2$, in (4) are the mass numbers, the equivalent sharp radii, and Yukawa diffuseness parameters a, respectively, of target and projectile. Expression (4) has also been derived in [9]. Various approximations to (4) are discussed in these papers.

In Figs. 1 and 2 the heavy-ion potentials for the systems $^{110}Pd + {}^{40}Ar$ and $^{12}C + {}^{12}C$, respectively, have been calculated with Woods–Saxon shapes for the density distribution and with folded Yukawa shapes. For numerical purposes integral (1) was converted into a discrete Fourier transform by the substitution $q \to x = 2rq/\pi - 2n - 1$ [7]

$$\int_0^\infty h(q) \sin qr\, dr = \frac{\pi}{2r} \int_{-1}^{1} \left[\sum_{n=1}^{\infty} (-)^n h\left(\frac{\pi}{2r} \{x + 2n + 1\}\right) \right] \cos \frac{\pi}{2} x\, dx$$

with

$$h(q) = F[\rho_P; q]\, F[\rho_T; q] \frac{q}{2\pi^2 r} \sum_{i=1}^{3} v_i F[Y_i; q].$$

The discrete Fourier integral was evaluated by a 96-point Gauss–Legendre formula. It is seen in the figures that the difference between the use of Woods–Saxon shapes and folding products is only noticeable in the tail of the potential. For comparison asymptotic expressions (3) and (4) are also shown in the figures.

A standard parameter set for the equivalent sharp radius R and diffuseness b of the densities was used [8]

$$R = (1.28A^{1/3} - 0.76 + 0.8A^{-1/3})\ \text{fm},$$
$$b = 1.0\ \text{fm}.$$

To avoid numerical complications in (3) and (4) the projectile diffuseness was taken to be 1% different from the target diffuseness.

In the figures, $(\cdots)$ represents a smooth parabolic extrapolation of (4) for $r < R_1 + R_2$. Its value for $r = 0$ was taken to be the fusion reaction Q-value minus the difference of the Coulomb self-energies of the compound nucleus and the two nuclei in the entrance channel. It is interesting that folding product (1) leads to an even stronger attraction in the interior although the underlying frozen–density model implies a doubling of the density for $r = 0$. Realistically this should give rise to a strong repulsion if the M3Y potential were prima facie a G matrix including a mock-up for exchange and kinetic energy contributions as implied by (1). This is obviously not the case so that the interior may be better described by a smooth interpolation like $(\cdots)$ in the figures.

Numerically folding potentials derived from densities with standard diffuseness b obey with fairly high accuracy the scaling law

$$V(R_1 + R_2 + s)/V(R_1 + R_2) = f(s/b), \qquad s \geqslant 0,$$

where R_1 and R_2 are equivalent sharp radii and $f(x)$ is a universal function. In optical-model calculations this may be used to reduce the computing time by a spline representation of $f(x)$.

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